

## Unit C2

# Core Mathematics 2

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

## C2.1 Unit description

Algebra and functions; coordinate geometry in the  $(x, y)$  plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration.

## C2.2 Assessment information

### Prerequisites

A knowledge of the specification for C1, its preamble and its associated formulae, is assumed and may be tested.

### Examination

The examination will consist of one 1½ hour paper. It will contain about nine questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

### Calculators

Students are expected to have available a calculator with at least the following keys:  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $\pi$ ,  $x^2$ ,  $\sqrt{x}$ ,  $\frac{1}{x}$ ,  $x^y$ ,  $\ln x$ ,  $e^x$ ,  $x!$ , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

**Formulae**

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

**Laws of logarithms**

$$\log_a x + \log_a y = \log_a(xy)$$

$$\log_a x - \log_a y \equiv \log_a \left( \frac{x}{y} \right)$$

$$k \log_a x = \log_a(x^k)$$

**Trigonometry**

In the triangle  $ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{area} = \frac{1}{2} ab \sin C$$

**Area**

$$\text{area under a curve} = \int_a^b y \, dx \quad (y \geq 0)$$

## 1 Algebra and functions

### What students need to learn:

Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.

Only division by  $(x + a)$  or  $(x - a)$  will be required.

Students should know that if  $f(x) = 0$  when  $x = a$ , then  $(x - a)$  is a factor of  $f(x)$ .

Students may be required to factorise cubic expressions such as  $x^3 + 3x^2 - 4$  and  $6x^3 + 11x^2 - x - 6$ .

Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial  $f(x)$  is divided by  $(ax + b)$ .

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## 2 Coordinate geometry in the $(x, y)$ plane

### What students need to learn:

Coordinate geometry of the circle using the equation of a circle in the form  $(x - a)^2 + (y - b)^2 = r^2$  and including use of the following circle properties:

- (i) the angle in a semicircle is a right angle;
- (ii) the perpendicular from the centre to a chord bisects the chord;
- (iii) the perpendicularity of radius and tangent.

Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.

### 3 Sequences and series

#### What students need to learn:

The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of  $|r| < 1$ .

Binomial expansion of  $(1 + x)^n$  for positive integer  $n$ .

The notations  $n!$  and  $\binom{n}{r}$ .

The general term and the sum to  $n$  terms are required.

The proof of the sum formula should be known.

Expansion of  $(a + bx)^n$  may be required.

### 4 Trigonometry

#### What students need to learn:

The sine and cosine rules, and the area of a triangle in the form  $\frac{1}{2} ab \sin C$ .

Radian measure, including use for arc length and area of sector.

Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

Knowledge and use of  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ , and  $\sin^2 \theta + \cos^2 \theta = 1$ .

Use of the formulae  $s = r\theta$  and  $A = \frac{1}{2} r^2 \theta$  for a circle.

Knowledge of graphs of curves with equations such as

$y = 3 \sin x$ ,  $y = \sin \left( x + \frac{\pi}{6} \right)$ ,  $y = \sin 2x$  is expected.

Solution of simple trigonometric equations in a given interval.

Students should be able to solve equations such as

$$\sin\left(x + \frac{\pi}{2}\right) = \frac{3}{4} \text{ for } 0 < x < 2\pi,$$

$$\cos(x + 30^\circ) = \frac{1}{2} \text{ for } -180^\circ < x < 180^\circ,$$

$$\tan 2x = 1 \text{ for } 90^\circ < x < 270^\circ,$$

$$6 \cos^2 x + \sin x - 5 = 0, 0^\circ \leq x < 360,$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2} \text{ for } -\pi \leq x < \pi.$$

## 5 Exponentials and logarithms

### What students need to learn:

$y = a^x$  and its graph.

Laws of logarithms

To include

$$\log_a xy = \log_a x + \log_a y,$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y,$$

$$\log_a x^k = k \log_a x,$$

$$\log_a \frac{1}{x} = -\log_a x,$$

$$\log_a a = 1$$

The solution of equations of the form  $a^x = b$ .

Students may use the change of base formula.

## 6 Differentiation

### What students need to learn:

Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.

The notation  $f''(x)$  may be used for the second order derivative.

To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

## 7 Integration

### What students need to learn:

Evaluation of definite integrals.

Interpretation of the definite integral as the area under a curve.

Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines.

Eg find the finite area bounded by the curve  $y = 6x - x^2$  and the line  $y = 2x$ .

$\int x \, dy$  will not be required.

Approximation of area under a curve using the trapezium rule.

For example,

evaluate  $\int_0^1 \sqrt{2x+1} \, dx$

using the values of  $\sqrt{2x+1}$  at  $x = 0, 0.25, 0.5, 0.75$  and  $1$ .