Unit FP1 Further Pure Mathematics 1

GCE AS and GCE Further Mathematics and GCE Pure Mathematics compulsory unit

FP1.1 Unit description

Series; complex numbers; numerical solution of equations; Coordinate systems, matrix algebra, proof.

FP1.2 Assessment information

Prerequisites	A knowledge of the specifications for C1 and C2, their prerequisites, preambles and associated formulae is assumed and may be tested.
Examination	The examination will consist of one 1½ hour paper. It will contain about nine questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted. Questions will be set in SI units and other units in common usage.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1 Complex numbers

What students need to learn:

Definition of complex numbers in the form $a + \mathrm{i}b$ and $r\cos\theta + \mathrm{i}\,r\sin\theta$.	The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.
Sum, product and quotient of complex numbers.	$\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \, \lvert z_2 \rvert$ Knowledge of the result $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ is not required.
Geometrical representation of complex numbers in the Argand diagram.	
Geometrical representation of sums, products and quotients of complex numbers.	
Complex solutions of quadratic equations with real coefficients.	
Conjugate complex roots of polynomial equations with real coefficients.	Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then z_1^* is also a root.

2 Numerical solution of equations

What students need to learn:

Equations of the form $\mathrm{f}(x) = 0$ solved numerically by: (i) interval bisection, (ii) linear interpolation, (iii) the Newton-Raphson process.	$\mathrm{f}(x)$ will only involve functions used in C1 and C2. For the Newton-Raphson process, the only differentiation required will be as defined in unit C1.

3 Coordinate systems

What students need to learn:

Cartesian equations for the parabola and rectangular hyperbola.	Students should be familiar with the equations: $y^2 = 4ax$ or $x = at^2$, $y = 2at$ and $xy = c^2$ or $x = ct$, $y = \frac{c}{t}$.
Idea of parametric equation for parabola and rectangular hyperbola.	The idea of $(at^2, 2at)$ as a general point on the parabola is all that is required.
The focus-directrix property of the parabola.	Concept of focus and directrix and parabola as locus of points equidistant from focus and directrix.
Tangents and normals to these curves.	Differentiation of $y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}$, $y = \frac{c^2}{x}$. Parametric differentiation is not required.

4 Matrix Algebra

What students need to learn:

Linear transformations of column vectors in two dimensions and their matrix representation.	The transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$.
Addition and subtraction of matrices.	
Multiplication of a matrix by a scalar.	
Products of matrices.	

Evaluation of 2 × 2 determinants.	Singular and non-singular matrices.
Inverse of 2 × 2 matrices.	Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.
Combinations of transformations.	Applications of matrices to geometrical transformations. Identification and use of the matrix representation of single and combined transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation of multiples of 45° about (0, 0) and enlargement about centre (0, 0), with scale factor, $(k \neq 0)$, where $k \in \mathbb{R}$.
The inverse (when it exists) of a given transformation or combination of transformations.	Idea of the determinant as an area scale factor in transformations.

5 Series

What students need to learn:

Summation of simple finite series.	Students should be able to sum series such as $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r(r^2+2)$. The method of differences is not required.

6 Proof

What students need to learn:

Proof by mathematical induction.	To include induction proofs for (i) summation of series eg show $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ or $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$ (ii) divisibility eg show $3^{2n} + 11$ is divisible by 4. (iii) finding general terms in a sequence eg if $u_{n+1} = 3u_n + 4$ with $u_1 = 1$, prove that $u_n = 3^n - 2$. (iv) matrix products eg show $\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$.

