Unit FP2 Further Pure Mathematics 2

GCE Further Mathematics and GCE Pure Mathematics A2 optional unit

FP2.1 Unit description

Inequalities; series, first order differential equations; second order differential equations; further complex numbers, Maclaurin and Taylor series.

FP2.2 Assessment information

Prerequisites	A knowledge of the specifications for C1, C2, C3, C4 and FP1, their prerequisites, preambles and associated formulae is assumed and may be tested.
Examination	The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted. Questions will be set in SI units and other units in common usage.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1 Inequalities

What students need to learn:

The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.	The solution of inequalities such as $\frac{1}{x-a} > \frac{x}{x-b}$ $\lvert x^2 - 1 \rvert > 2(x + 1)$.

2 Series

What students need to learn:

Summation of simple finite series using the method of differences.	Students should be able to sum series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ by using partial fractions such as $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$.

3 Further Complex Numbers

What students need to learn:

Euler's relation $e^{i\theta} = \cos\theta + i\sin\theta$.	Students should be familiar with $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$.
De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.	To include finding $\cos n\theta$ and $\sin m\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ and also powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles. Students should be able to prove De Moivre's theorem for any integer n.
Loci and regions in the Argand diagram.	Loci such as $\lvert z - a\rvert = b$, $\lvert z - a\rvert = k\lvert z - b\rvert$, $\arg(z - a) = \beta$, $\arg\frac{z-a}{z-b} = \beta$ and regions such as $\lvert z - a\rvert \le \lvert z - b\rvert$, $\lvert z - a\rvert \le b$.
Elementary transformations from the z-plane to the w-plane.	Transformations such as $w = z^2$ and $w = \frac{az+b}{cz+d}$, where $a, b, c, d \in \mathbb{C}$, may be set.

4 First Order Differential Equations

What students need to learn:

Further solution of first order differential equations with separable variables.	The formation of the differential equation may be required. Students will be expected to obtain particular solutions and also sketch members of the family of solution curves.
First order linear differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + Py = Q$ where P and Q are functions of x.	The integrating factor $e^{\int P \mathrm{d}x}$ may be quoted without proof.
Differential equations reducible to the above types by means of a given substitution.	

5 Second Order Differential Equations

What students need to learn:

The linear second order differential equation $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = \mathrm{f}(x)$ where a, b and c are real constants and the particular integral can be found by inspection or trial.	The auxiliary equation may have real distinct, equal or complex roots. $\mathrm{f}(x)$ will have one of the forms ke^{px}, $A + Bx$, $p + qx + cx^2$ or $m \cos \omega x + n \sin \omega x$. Students should be familiar with the terms 'complementary function' and 'particular integral'. Students should be able to solve equations of the form $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin 2x$.
Differential equations reducible to the above types by means of a given substitution.	

6 Maclaurin and Taylor series

What students need to learn:

Third and higher order derivatives.

Derivation and use of Maclaurin series.

The derivation of the series expansion of e^x, $\sin x$, $\cos x$, $\ln(1 + x)$ and other simple functions may be required.

Derivation and use of Taylor series.

The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x - \pi)$ up to and including the term in $(x - \pi)^3$.

Use of Taylor series method for series solutions of differential equations.

Students may, for example, be required to find the solution in powers of x as far as the term in x^4, of the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,$$

such that $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

7 Polar Coordinates

What students need to learn:

Polar coordinates (r, θ), $r \geq 0$.

The sketching of curves such as

$\theta = \alpha$, $r = p\sec(\alpha - \theta)$, $r = a$,

$r = 2a\cos\theta$, $r = k\theta$, $r = a(1 \pm \cos\theta)$,

$r = a(3 + 2\cos\theta)$, $r = a\cos 2\theta$ and

$r^2 = a^2\cos 2\theta$ may be set.

Use of the formula $\frac{1}{2}\int_\alpha^\beta r^2 \, d\theta$ for area.

The ability to find tangents parallel to, or at right angles to, the initial line is expected.

